

## Exercise Set IV, Computational Complexity 2018

These exercises are for your own benefit. Feel free to collaborate and share your answers with other students. Solve as many problems as you can and ask for help if you get stuck for too long. Problems marked \* are more difficult but also more fun :).

These problems are taken from various sources at EPFL and on the Internet, too numerous to cite individually.

- 1 (\*) This exercise is about the largeness condition of natural proofs. Prove that a random function  $g : \{0, 1\}^n \rightarrow \{0, 1\}$  satisfies  $\mathcal{P}(g) = 1$  with high probability, where  $\mathcal{P}$  is the property that no fixing of  $n - n^\epsilon$   $g$ 's inputs turns  $g$  into a constant function.
- 2 Show that  $\mathbf{dIP} = \mathbf{NP}$ . In other words, interactive proofs with polynomial number of interactions does not increase the set of languages we can decide if the verifier is deterministic.
- 3 Let  $\mathbf{IP}'$  denote the class obtained by allowing also the prover to be probabilistic. That is, the prover's strategy can be chosen at random from some distribution on functions. Prove that  $\mathbf{IP}' = \mathbf{IP}$ .
- 4 Let  $\mathbf{IP}''$  denote the class obtained by changing the constant  $1/3$  in the soundness guarantee of  $\mathbf{IP}$  to  $0$ . Prove that  $\mathbf{IP}'' = \mathbf{NP}$ .
- 5 (\*) Prove that  $\mathbf{IP} \subseteq \mathbf{PSPACE}$ . Recall that  $\mathbf{PSPACE}$  contain those languages recognizable by a TM that uses polynomial amount of space.