ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Homework IV, Topics in Theoretical Computer Science 2015

Due on Friday May 8 at 12:00 AM (send an email to ola.svensson@epfl.ch). Solutions to many homework problems, including problems on this set, are available on the Internet, either in exactly the same formulation or with some minor perturbation. It is *not acceptable* to copy such solutions. It is hard to make strict rules on what information from the Internet you may use and hence whenever in doubt contact Ola Svensson. You are, however, allowed to discuss problems in groups with up to three students.

1 *(Perfect Matchings, 55 pts)* Let G be a cubic graph, i.e., all vertices have degree 3.

1a *(15 pts)* Prove that if G is 2-edge-connected then it has perfect matching.

1b *(15 pts)* Prove that if G is 2-edge-connected then, for all edges e and f, $G \setminus \{e, f\}$ has a perfect matching.

(Here $G \setminus \{e, f\}$ denotes the subgraph of G where we removed edges e and f.)

1c *(25 pts)* Deduce that if G has a unique bridge (that is a unique cut of a single edge, and all other cuts have size at least 2) then G has a perfect matching.

2 *(Ellipsoid method, 45 pts)* The ellipsoid method is a very powerful method not limited to linear programming. An important larger class of problems where it is useful is semidefinite programming. In semidefinite programming, we have n^2 variables $\{X_{ij}\}_{1\leq i,j\leq n}$ together with m linear constraints (as in linear programming).In addition, we have the constraint that the matrix formed by the X_{ij}'s is positive semidefinite. If we let X denote the matrix whose (i, j) entry equals X_{ij}, we have that the convex set of feasible points is defined by

$$\sum_{1\leq i,j\leq n} a^{\ell}_{ij} X_{ij} \leq b^{\ell} \qquad \text{for } \ell = 1, 2, \ldots, m \qquad \textit{(the } m \textit{ linear constraints)}$$

$$X \succeq 0 \qquad \textit{(the matrix } X \textit{ is positive semidefinite)}$$

2a *(25 pts)* Devise an efficient separation oracle (polynomial in n and m) for deciding whether a given $X^* = \{X^*_{ij}\}_{1\leq i,j\leq n}$ is feasible, and if not, you should output a separating hyperplane.

2b *(20 pts)* Does the efficient separation oracle imply that we can use the ellipsoid method to decide whether a given semidefinite program is feasible or not? *Motivate your answer.*