ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Homework I, Approximation Algorithms and Hardness of Approximation 2013

Due on Tuesday March 19 at 16.15 (send an email to ola.svensson@epfl.ch). Solutions to many homework problems, including problems on this set, are available on the Internet, either in exactly the same formulation or with some minor perturbation. It is *not acceptable* to copy such solutions. It is hard to make strict rules on what information from the Internet you may use and hence whenever in doubt contact Ola Svensson. You are, however, allowed to discuss problems in groups with up to three students, but solutions should be handed in individually and please write with whom you have collaborated.

On this problem set there are four problems. Although, we recommend you to solve all four, we will grade Problems 1, 2 and one out of Problems 3 and 4 for credit, and the total number of points you can receive is 100. Therefore, clearly state which of Problems 3 and 4 you want graded for credit.

1 (30 p, problem 2.1 (a) in the book of Williamson and Shmoys) The *k-suppliers* problem is similar to the k-center problem that we saw in class. The input to the problem is a positive integer k, and a set of vertices V, along with non-negative distances d_{ij} between any two vertices i, j that are symmetric and obey the triangle inequality. However, now the vertices are partitioned into *suppliers* $F \subseteq V$ and *customers* $D = V \setminus F$. The goal is to find k suppliers such that the maximum distance from a supplier to a customer is minimized. In other words, we wish to find $S \subseteq F, |S| \leq k$, that minimizes $\max_{j \in D} d(j, S)$.

Give a 3-approximation algorithm for the k-supplier problem.

2 (30 p) In class, we saw that "naive" list scheduling gave a 2-approximation algorithm for makespan minimization on identical machines. We then improved this, by using the LPT (Largest remaining Processing Time) rule, to a 4/3-approximation algorithm.

In this problem, we shall analyze the performance of list scheduling for the same problem when we also have precedence constraints between jobs. Specifically, given a number m of machines, a set $\mathcal{J} = \{1, \ldots, n\}$ of n jobs with processing times $p_1, p_2, \ldots, p_n$, and precedence constraints between jobs in the form of a partial order $(\mathcal{J}, <)$ on the jobs, find the shortest schedule that complies with the precedence constraints, i.e., if $j < j'$ then job j has to be completed before job j' can be started.

2a (20 p) Show that naive list scheduling yields a 2-approximation algorithm for this problem.

2b (10 p) Give a class of instances that only consists of unit-time jobs and for which the list scheduling algorithm may return schedules whose lengths tend to $2 \cdot OPT$ as n and m tends to infinity.

(This shows that a LPT rule is not applicable for the problem with precedence constraints. In fact it is hard to do better than 2 assuming the so-called unique games conjecture.)

Select one of Problems 3 and 4 to be graded (although we recommend you to do both)

3 (40 p, problem 9.7 in the book of Vazirani) *Bin covering* is the problem where we are given n items with sizes $a_1, \ldots, a_n \in (0, 1]$ and we wish to maximize the number of bins opened so that each bin has items summing up to at least 1.

Give an asymptotic PTAS for this problem when restricted to instances in which item sizes are bounded below by c, for a fixed constant $c > 0$.

4 (40 p) For the *Euclidean k-TSP* problem, we are given points $v_1, \ldots, v_n \in \mathbb{Q}^2$ in the plane and a parameter $k \in \{1, \ldots, n\}$. The goal is to find a minimum length tour, *visiting at least k nodes*. Here the length is measured using the Euclidean distances. Give a PTAS for this problem (by adapting Arora's algorithm for Euclidean TSP).