Homework II, Approximation Algorithms and Hardness of Approximation 2013

Due on Tuesday April 9 at 16.15 (send an email to ola.svensson@epfl.ch). Solutions to many homework problems, including problems on this set, are available on the Internet, either in exactly the same formulation or with some minor perturbation. It is *not acceptable* to copy such solutions. It is hard to make strict rules on what information from the Internet you may use and hence whenever in doubt contact Ola Svensson. You are, however, allowed to discuss problems in groups with up to three students, but solutions should be handed in individually and please write with whom you have collaborated.

1 (20 p, problem 14.6 in the book of Vazirani) We saw that any extreme point solution to the vertex cover LP is half-integral. This does not generalize to set cover. Indeed, give a counterexample to the following claim:

"A set cover instance in which each element is in exactly f sets has a $(1/f)$-integral optimal fractional solution (i.e., in which each set is picked an integer multiple of $1/f$)."

2 (20 p) In class we saw that the natural linear programming relaxation of the maximum weight bipartite matching problem is integral, i.e., every extreme point solution is integral. Use this fact to prove the following classical result:

"The edge set of a k-regular bipartite graph $G = (V_1 \cup V_2, E)$ can in polynomial time be partitioned into k-disjoint matchings".

A graph is k-regular if the degree of each vertex equals k. Two matchings are disjoint if they do not share any edges.

3 (30 p, problem 3.4 in the book of Lau, Ravi, and Singh) Given a set of intervals $[a_i, b_i]$ for each $1 \leq i \leq n$ and a weight function w on intervals, the *maximum weight k-interval packing* problem asks for a subset J of intervals of maximum weight such that there are at most k intervals in J at any point on the line.

(a) Formulate a linear program for the maximum weight k-interval packing problem.

(b) Show that only $n-1$ point constraints need to be imposed apart from bound constraints (that is constraints of the form $0 \leq x_i \leq 1$).

(c) Show that the linear program is integral.

4 (30 p) In a Vertex Cover instance $G = (V, E)$ an edge $u, v \in E$ equals the constraint that either u or v must be picked. For some applications it is undesirable to pick both so we need to also introduce a second type, called exclusive-or edges, that requires us to pick exactly one of the two incident vertices. Give a polynomial time algorithm for the generalization of Vertex Cover, where we have both ordinary and exclusive-or edges, that verifies if a solution exists and if it exists returns a 2-approximate solution.