ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Homework IV, Approximation Algorithms and Hardness of Approximation 2013

Due on Tuesday May 14 at 16.15 (send an email to ola.svensson@epfl.ch). Solutions to many homework problems, including problems on this set, are available on the Internet, either in exactly the same formulation or with some minor perturbation. It is *not acceptable* to copy such solutions. It is hard to make strict rules on what information from the Internet you may use and hence whenever in doubt contact Ola Svensson. You are, however, allowed to discuss problems in groups with up to three students, but solutions should be handed in individually and please write with whom you have collaborated. On this problem set, Problem 3b is about writing the alphabet reduction part of the proof of the PCP theorem in your own words and here you should feel free to use any source.

1 (30 pts) Let $G = (V, E)$ be a 3-colorable graph. Give a polynomial time algorithm that picks a subset of the vertices of size at least $n/2$ such that the subgraph induced on this subset does not contain any triangles. Prove the correctness of your algorithm.

2 (30 pts, Problem 11.8 in the book of Arora & Barak) Show that if $SAT \in \mathbf{PCP}(r(n), 1)$ for some $r(n) = o(\log n)$ then $\mathbf{P} = \mathbf{NP}$. This shows that the PCP theorem is probably optimal up to constants.

Hint: Consider first the case when $r(n) = \log \log n$.

3 (40 pts) The two following subproblems will prove the alphabet reduction part of the PCP theorem. For 3b you are allowed to use any source but you should write it in your own words.

3a (15 pts) Prove that QUADEQ is **NP**-complete by giving a reduction from CKT-SAT. Hint: The QUADEQ instance will have a variable for each wire in the circuit (including input wires).

The language circuit satisfiability or (CKT-SAT) consists of all circuits that produce a single bit of output and that have a satisfying assignment (i.e., an input so that the circuit outputs 1). (A circuit consists of the logical operators OR, AND, and NOT; OR and AND have fan-in exactly equal to 2 and NOT has fan-in exactly equal to 1.)

3b Prove Lemma 22.6 in the book of Arora & Barak: There exists a constant q_0 and a CL-reduction h such that for every CSP instance ϕ, if ϕ had arity two over a (possibly nonbinary) alphabet $\{0, \ldots, W - 1\}$ then $h(\phi)$ has arity q_0 over a binary alphabet and satisfies:

$$val(\phi) \leq 1 - \epsilon \Rightarrow val(h(\phi)) \leq 1 - \epsilon/3.$$

For this exercise you are allowed to use any source but you should write the solution in your own words.